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Multi-Photon Amplitudes for Next-to-Leading Order QCD

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Abstract

We present the tree-level amplitudes involving one, two and three photons that are required for next-to-leading order QCD calculations of production rates of three final-state particles. We also present the required one-loop amplitudes in terms of previously published results.

1 Introduction

The primary goal of the LHC physics program is the investigation of the mechanism of electroweak symmetry breaking; to observe the Higgs boson, if it exists, and to study its properties. If the mass of the Higgs is light (100 GeV $\leq m_H \leq$ 140 GeV), studies indicate that the rare decay into two photons, $H \to \gamma\gamma$ provides the best signature [1]. Because the signal-to-background ratio will be quite low ($\sim 7\%$) [1], this promises to be a demanding analysis. Unfortunately, our theoretical understanding of both the signal and the background is less than satisfactory. The next-to-leading order (NLO) QCD corrections to Higgs production are known to be very large ($\mathcal{O}(100\%)$) [2]. The NLO corrections to the QCD background [3], $pp \to \gamma\gamma$, (for which the full NLO fragmentation contributions are just now being evaluated [4]) are known to also be very large, and there are strong indications that the next-to-next-to-leading order (NNLO) corrections will also be sizable because of the large gluon luminosity at LHC which will enhance the $gg \to \gamma\gamma$ subprocess [5, 6] that first appears at NNLO. Thus it seems that we will need (at least) full NNLO calculations of both the signal and the background in order to obtain a reasonable theoretical understanding of this process.

In order to improve the signal-to-background ratio, Higgs production in association with a high transverse energy (E_T) jet, $pp \to H\ jet \to \gamma\gamma\, jet$, has been considered [7]. This process offers the advantage of being more flexible with respect to choosing suitable acceptance cuts to curb the background. The $pp \to H\ jet$ process is known to leading order (LO) exactly [8], while the NLO calculation [9] has been performed in the infinite top mass limit. The NLO corrections to the signal are also large, but it is believed that the background, $pp \to \gamma\gamma\, jet$, can be more reliably calculated because LO production is dominated by the $qg \to q\gamma\gamma$ sub-process, which benefits from the large gluon luminosity, while the $gg \to g\gamma\gamma$ sub-process, which is believed to dominate the NNLO contribution, yields a comparatively small contribution [5]. Thus an evaluation of the full NLO corrections to the background should provide a reliable quantitative estimate.

In order to evaluate the NLO corrections to $pp \to \gamma\gamma\, jet$, we need tree-level QCD amplitudes with six external legs ($\bar{q}qgg\gamma\gamma$, $\bar{q}q\overline{Q}Q\gamma\gamma$) and one-loop amplitudes with five external legs ($\bar{q}qg\gamma\gamma$). The same amplitudes also contribute to the evaluation of the NNLO corrections to $pp \to \gamma\gamma$ scattering. Of course, there are other processes involving one or more photons that can be computed at NLO, including the recently observed direct photo-production of three jets at HERA [10].

In principle, the amplitudes required for the calculations mentioned above can be obtained from pure QCD amplitudes by turning external gluons into photons. This is done by summing over permutations of the color-ordered amplitudes [11] to remove the non-Abelian character of the gluon coupling (see secs. 2.2,6.2). This is the procedure we use to obtain the one-loop amplitudes. Of course, this procedure naturally results in more complicated expressions than those for the pure QCD amplitudes with which one starts. One generally obtains far more compact expressions by computing the photon amplitudes

directly, and this is what we do for the tree-level amplitudes.

In this paper, we present all of the matrix elements needed to compute at NLO two-parton to three-parton scattering where one, two or three of the partons are photons. These amplitudes, with the one-loop amplitudes supplemented by higher-order terms in the dimensional-regularization parameter ϵ [12], also contribute to the computation at NNLO of two-parton to two-parton scattering where one or two of the partons are photons. Many of these amplitudes have been discussed previously. The five-parton tree-level amplitudes involving photons have been known for some time [13, 14]. The five-parton one-loop amplitudes involving one photon have been discussed in detail [15, 16] and are reproduced here for completeness. The new results presented here are the compact expressions for the six-parton tree-level amplitudes involving photons and the detailed expansions for the five-parton one-loop multi-photon amplitudes.

The paper is organized as follows: in section 2 we present the known results for the amplitudes at tree level and explain the rules which allow one to turn external gluons into photons. In sections 3, 4 and 5, we evaluate the six-parton tree-level amplitudes with one, two and three photons, respectively. In section 6, we present detailed expansions for the corresponding five-parton one-loop amplitudes in terms of known results. In section 7 we draw our conclusions.

2 Tree-level Amplitudes

It is now common to express multi-parton amplitudes in QCD using a color-ordered/helicity decomposition [11]. The usual color-ordered decomposition follows from replacing the group structure constants with commutators of fundamental representation matrices [1]

$$f^{abc} = -\frac{i}{\sqrt{2}}(\mathrm{Tr}[T^aT^bT^c] - \mathrm{Tr}[T^bT^aT^c]) \tag{2.1}$$

and then using $SU(N_c)$ Fierz identities

$$T^a_{i\bar{j}}\,T^a_{m\bar{n}} = \delta_{i\bar{n}}\,\delta_{m\bar{j}} - \frac{1}{N_c}\,\delta_{i\bar{j}}\,\delta_{m\bar{n}} \tag{2.2}$$

to combine traces. Each distinct ordering of representation matrices (Chan-Paton, or color factor) is associated with a gauge invariant sub-amplitude depending only on the ordering of the parton labels. The full amplitude is expressed as the sum over the independent color factors multiplying their associated sub-amplitude.

The helicity decomposition is accomplished by specifying the helicity of each external leg. Typically, helicity amplitudes are written in terms of spinor products $\langle i\,j\rangle$, $[i\,j]$. The spinor products are antisymmetric, with norm $|\,\langle i\,j\rangle\,| = |\,[i\,j]\,| = \sqrt{|s_{ij}|}$. We use the convention of Mangano and Parke [14], defining $\langle i\,j\rangle = \langle i^-|j^+\rangle$ and $[i\,j] = \langle i^+|j^-\rangle$, where

[1]We normalize the fundamental representation matrices such that $\mathrm{Tr}[T^aT^b] = \delta^{ab}$.

$|i^{\pm}\rangle$ are massless Weyl spinors of momentum k_i, labeled with the sign of the helicity. In this convention, the spinor products obey the rule $\langle i\, j\rangle\, [j\, i] = s_{ij} = 2k_i \cdot k_j$.

2.1 Two-quark amplitudes

Tree-level amplitudes involving two quarks and $(n-2)$ gluons are written as [11, 14],

$$\mathcal{A}_n^{\text{tree}}(\bar{q}_1, q_2; g_3, \ldots, g_n) = g^{n-2} \sum_{\sigma \in S_{n-2}} (T^{\sigma_3} \ldots T^{\sigma_n})_{i_2}^{\bar{\imath}_1}\; A_n^{\text{tree}}(\bar{q}_1, q_2, g_{\sigma_3}, \ldots, g_{\sigma_n})\,, \tag{2.3}$$

where S_{n-2} is the permutation group on the $n-2$ elements $(3 \ldots n)$. The dependence on parton helicities and momenta is implicit.

For the *maximally-helicity-violating* configurations, in which two partons are of one helicity and all others are of the opposite helicity (where helicity is defined as if all particles are outgoing), say $(-,-,+,\ldots,+)$, there is only one independent color/helicity sub-amplitude

$$A_n^{\text{tree}}(\bar{q}_1^+, q_2^-, g_3, \ldots, g_n) = i\frac{\langle 1\, i\rangle \langle 2\, i\rangle^3}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle n\, 1\rangle} \tag{2.4}$$

where gluon g_i has negative helicity. (The quark and anti-quark must have opposite helicity in order to conserve helicity along the fermion line.)

All other color/helicity amplitudes can be obtained by relabeling and by use of the discrete symmetries of parity inversion and charge conjugation. Parity inversion flips the helicities of all particles. It is accomplished by the "complex conjugation" operation (indicated with a superscript †) defined such that $\langle i\, j\rangle \leftrightarrow [j\, i]$, but explicit factors of i are not conjugated to $-i$. In addition, there is a factor of -1 for each pair of quarks participating in the amplitude. Charge conjugation changes quarks into anti-quarks without inverting helicities. In addition, there is a reflection symmetry in the color ordering such that

$$A_n^{\text{tree}}(\bar{q}_1^+, q_2^-, g_3, \ldots, g_n) = (-1)^n A_n^{\text{tree}}(q_2^-, \bar{q}_1^+, g_n, \ldots, g_3) \tag{2.5}$$

So, if gluon i has negative helicity and all other gluons have positive helicity, one finds the sub-amplitude $A_n^{\text{tree}}(\bar{q}_1^-, q_2^+, g_3, \ldots, g_n)$ to be

$$A_n^{\text{tree}}(\bar{q}_1^-, q_2^+, g_3, \ldots, g_n) = (-1)^n A_n^{\text{tree}}(\bar{q}_2^+, q_1^-, g_n, \ldots, g_3) = i\frac{\langle 1\, i\rangle^3 \langle 2\, i\rangle}{\langle 1\, 2\rangle \langle 2\, 3\rangle \ldots \langle n\, 1\rangle}. \tag{2.6}$$

2.2 Turning gluons into photons

The color decomposition of amplitudes involving two quarks, m photons and r gluons, with $m + r = n - 2$, is

$$\begin{aligned} &\mathcal{A}_n^{\text{tree}}(\bar{q}_1, q_2; g_3, \ldots, g_{r+2}; \gamma_{r+3}, \ldots, \gamma_n) = \\ &\quad g^r\, (\sqrt{2}\, eQ_q)^m \sum_{\sigma \in S_r} (T^{\sigma_3} \ldots T^{\sigma_{r+2}})_{i_2}^{\bar{\imath}_1}\; A_n^{\text{tree}}(\bar{q}_1, q_2, g_{\sigma_3}, \ldots, g_{\sigma_{r+2}}, \gamma_{r+3}, \ldots, \gamma_n)\,. \end{aligned} \tag{2.7}$$

The color-ordered amplitudes can be computed directly or can be obtained from two-quark $(n-2)$ gluon amplitudes by converting gluons into photons. In order to convert a gluon into a photon we replace the quark-gluon vertex factor gT^a with the quark-photon vertex factor[2] $\sqrt{2}\,eQ_qI$. Since the identity matrix I commutes with the $SU(N)$ matrices T^a, all possible attachments of the photon to the quark line (leaving the gluon ordering fixed) contribute to the same color structure. That is, matrix elements involving photons are obtained by summing over permutations of gluon matrix elements in which the photon assumes all possible places in the ordering. For example,

$$\begin{aligned}
&A_5^{\text{tree}}(\bar{q}_1, q_2, g_3, g_4; \gamma_5) = \\
&\qquad A_5^{\text{tree}}(\bar{q}_1, q_2, g_3, g_4, g_5) + A_5^{\text{tree}}(\bar{q}_1, q_2, g_3, g_5, g_4) + A_5^{\text{tree}}(\bar{q}_1, q_2, g_5, g_3, g_4).
\end{aligned} \tag{2.8}$$

For maximally-helicity-violating configurations, $(-,-,+,\ldots,+)$, the sub-amplitudes are found to be [11, 14],

$$A_n^{\text{tree}}(\bar{q}_1^+, q_2^-, g_3, \ldots, g_{r+2}, \gamma_{r+3}, \ldots, \gamma_{r+m+2}) = i\frac{\langle 1\, i\rangle \langle 2\, i\rangle^3}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle (r+2)\, 1\rangle} \prod_{j=r+3}^{r+m+2} \left[\frac{\langle 2\, 1\rangle}{\langle 2\, j\rangle \langle j\, 1\rangle}\right] \tag{2.9}$$

with i the negative helicity gluon or photon. The term associated with each photon is simply the eikonal factor written in spinor-helicity notation,

$$\frac{\langle 2\, 1\rangle}{\langle 2\, j\rangle \langle j\, 1\rangle} = \sqrt{2}\left[\frac{\varepsilon_j^+ \cdot k_1}{s_{1j}} - \frac{\varepsilon_j^+ \cdot k_2}{s_{j2}}\right]. \tag{2.10}$$

2.3 Four-quark amplitudes

For four quarks and $(n-4)$ gluons the color decomposition of the tree-level amplitude is [14]

$$\begin{aligned}
&\mathcal{A}_n^{\text{tree}}(\bar{q}_1, q_2, \overline{Q}_3, Q_4, g_5, \ldots, g_n) = g^{n-2} \sum_{k=0}^{n-4} \sum_{\sigma \in S_k} \sum_{\rho \in S_l} \\
&\quad \times \left[(T^{\sigma_1} \ldots T^{\sigma_k})_{i_4}^{\bar{\imath}_1} \, (T^{\rho_1} \ldots T^{\rho_l})_{i_2}^{\bar{\imath}_3} \, A_n^{\text{tree}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1}, \ldots, g_{\sigma_k}; g_{\rho_1}, \ldots, g_{\rho_l}) \right. \\
&\quad - \left. \frac{1}{N_c} (T^{\sigma_1} \ldots T^{\sigma_k})_{i_2}^{\bar{\imath}_1} \, (T^{\rho_1} \ldots T^{\rho_l})_{i_4}^{\bar{\imath}_3} \, B_n^{\text{tree}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1}, \ldots, g_{\sigma_k}; g_{\rho_1}, \ldots, g_{\rho_l}) \right],
\end{aligned} \tag{2.11}$$

with $k + l = n - 4$. The sums are over the partitions of $(n-4)$ gluons between the two quark lines, and over the permutations of the gluons within each partition. For $k = 0$ or $l = 0$, the color strings reduce to Kronecker delta's. For identical quarks, we must subtract from this equation the same term with the quarks (but not the anti-quarks) exchanged $(q_2 \leftrightarrow Q_4)$.

[2] The factor $\sqrt{2}$ is due to our choice of normalization of the fundamental representation matrices.

For maximally-helicity-violating configurations, $(-,-,+,\ldots,+)$, where because of helicity conservation along the fermion lines all of the gluons have the same helicity, the $A_n^{\rm tree}$ and $B_n^{\rm tree}$ sub-amplitudes factorize into distinct contributions for the two quark antennæ [11, 14],

$$
\begin{aligned}
&A_n^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1},\ldots,g_{\sigma_k}; g_{\rho_1},\ldots,g_{\rho_l}) = \\
&\quad i\frac{f(\lambda_{q_2},\lambda_{Q_4})}{\langle \bar q_1\, q_2\rangle \left\langle \overline{Q}_3\, Q_4\right\rangle} \frac{\left\langle q_2\, \overline{Q}_3\right\rangle}{\langle q_2\, g_{\sigma_1}\rangle\cdots\left\langle g_{\sigma_k}\, \overline{Q}_3\right\rangle} \frac{\langle Q_4\, \bar q_1\rangle}{\langle Q_4\, g_{\rho_1}\rangle\cdots\langle g_{\rho_l}\, \bar q_1\rangle} \\
&B_n^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1},\ldots,g_{\sigma_k}; g_{\rho_1},\ldots,g_{\rho_l}) = \\
&\quad i\frac{f(\lambda_{q_2},\lambda_{Q_4})}{\langle \bar q_1\, q_2\rangle \left\langle \overline{Q}_3\, Q_4\right\rangle} \frac{\langle q_2\, \bar q_1\rangle}{\langle q_2\, g_{\sigma_1}\rangle\cdots\langle g_{\sigma_k}\, \bar q_1\rangle} \frac{\left\langle Q_4\, \overline{Q}_3\right\rangle}{\langle Q_4\, g_{\rho_1}\rangle\cdots\left\langle g_{\rho_l}\, \overline{Q}_3\right\rangle},
\end{aligned}
\tag{2.12}
$$

with

$$
\begin{aligned}
f(+,+) &= -\left\langle \bar q_1\, \overline{Q}_3\right\rangle^2 \qquad & f(+,-) &= \langle \bar q_1\, Q_4\rangle^2 \\
f(-,+) &= \left\langle q_2\, \overline{Q}_3\right\rangle^2 \qquad & f(-,-) &= -\langle q_2\, Q_4\rangle^2 .
\end{aligned}
\tag{2.13}
$$

This factorization does not occur for more complicated helicity configurations.

When we convert a gluon into a photon we attach the photon everywhere along the two quark lines, i.e. sum over all of the permutations of the photon index in the global color ordering, and we replace the color charge g with the appropriate quark electric charge $\sqrt{2}\, eQ_{q_i}$. The ensuing color decomposition of the amplitude for the emission of m photons and r gluons, with $m+r=n-4$, is

$$
\begin{aligned}
&\mathcal{A}_n^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_5,\ldots,g_{r+4}; \gamma_{r+5},\ldots,\gamma_n) = g^{r+2}\,(\sqrt{2}e)^m \sum_{k=0}^{r}\sum_{\sigma\in S_k}\sum_{\rho\in S_l} \\
&\times \Bigg[(T^{\sigma_1}\ldots T^{\sigma_k})_{i_4}^{\bar\imath_1}\, (T^{\rho_1}\ldots T^{\rho_l})_{i_2}^{\bar\imath_3} \\
&\qquad \times A_n^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1},\ldots,g_{\sigma_k}; g_{\rho_1},\ldots,g_{\rho_l}; \gamma_{r+5},\ldots,\gamma_n)) \\
&- \frac{1}{N_c}\, (T^{\sigma_1}\ldots T^{\sigma_k})_{i_2}^{\bar\imath_1}\, (T^{\rho_1}\ldots T^{\rho_l})_{i_4}^{\bar\imath_3} \\
&\qquad \times B_n^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1},\ldots,g_{\sigma_k}; g_{\rho_1},\ldots,g_{\rho_l}; \gamma_{r+5},\ldots,\gamma_n)) \Bigg],
\end{aligned}
\tag{2.14}
$$

with $k+l=r$. The dependence on the quark electric charges Q_p is contained in the sub-amplitudes $A_n^{\rm tree}$ and $B_n^{\rm tree}$. For maximally-helicity-violating configurations, $(-,-,+,\ldots,+)$, with all of the gluons and photons having positive helicity, the sub-amplitudes are [11, 14],

$$
\begin{aligned}
&A_n^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1},\ldots,g_{\sigma_k}; g_{\rho_1},\ldots,g_{\rho_l}; \gamma_{r+5},\ldots,\gamma_n) = \\
&\prod_{j=r+5}^{n}\left(\frac{Q_q\,\langle q_2\, \bar q_1\rangle}{\langle q_2\, \gamma_j\rangle\langle \gamma_j\, \bar q_1\rangle} + \frac{Q_Q\,\left\langle Q_4\, \overline{Q}_3\right\rangle}{\langle Q_4\, \gamma_j\rangle\left\langle \gamma_j\, \overline{Q}_3\right\rangle}\right) A_{n-m}^{\rm tree}(\bar q_1, q_2; \overline{Q}_3, Q_4; g_{\sigma_1},\ldots,g_{\sigma_k}; g_{\rho_1},\ldots,g_{\rho_l})
\end{aligned}
\tag{2.15}
$$

and the same for $B_n^{\rm tree}$.

3 One-Photon Amplitudes

One-photon amplitudes with five and six external legs can be used for NLO calculations of direct photo-production of three jets at electron-proton colliders [10] or photon + two-jet production at hadron colliders. They also contribute to NNLO calculations of the same processes with one less jet in the final state.

The tree-level amplitudes with five external legs involve only maximally-helicity-violating configurations and are easily obtained from the formulæ of the previous section.

The one-photon tree-level amplitudes with six external legs are

a) two-quark, three-gluon, one-photon amplitudes;

b) four-quark, one-gluon, one-photon amplitudes.

3.1 Two-quark, three-gluon, one-photon amplitudes

The color decomposition of the two-quark, three-gluon, one-photon amplitudes is given in eq. (2.7), with $r = 3$ and $m = 1$. For maximally-helicity-violating configurations, the sub-amplitudes are given in eq. (2.9). Making use of discrete symmetries, there are three distinct helicity configurations of type $(-,-,-,+,+,+)$, which are listed in table 1.

	$\bar{q}_5$	q_6	g_1	g_2	g_3	γ_4
I	$+$	$-$	$-$	$-$	$+$	$+$
II	$+$	$-$	$+$	$-$	$-$	$+$
III	$+$	$-$	$-$	$+$	$-$	$+$

Table 1: The distinct helicity configurations of type $(-,-,-,+,+,+)$, for the two-quark, three-gluon, one-photon amplitudes.

For these configurations, the sub-amplitudes can be written as

$$\begin{aligned} -\,iA_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^-, g_3^+, \gamma_4^+) \;=\;& \frac{\langle 3|(1+2)|5\rangle\langle 4|(3+5)|2\rangle t_{126}}{[1\,2]\langle 4\,5\rangle[1\,6]\langle 3\,5\rangle s_{23}s_{46}} + \frac{\langle 3|(2+5)|1\rangle\langle 4|(3+5)|2\rangle^2}{\langle 3\,5\rangle[1\,6]s_{23}s_{46}t_{146}} \\ &- \frac{\langle 1\,2\rangle[4\,5]\langle 3|(1+2)|6\rangle\langle 3|(1+2)|5\rangle}{[1\,2]\langle 4\,5\rangle s_{23}s_{46}t_{123}} \end{aligned} \tag{3.1}$$

$$\begin{aligned} -\,iA_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^+, g_2^-, g_3^-, \gamma_4^+) \;=\;& \frac{\langle 2\,3\rangle\langle 1|(2+3)|6\rangle^2}{[2\,3]\langle 5\,4\rangle\langle 4\,6\rangle s_{12}t_{123}} + \frac{\langle 2\,6\rangle\langle 1|(2+3)|6\rangle\langle 1|(2+6)|3\rangle t_{146}}{[2\,3]\langle 5\,4\rangle\langle 4\,6\rangle s_{12}s_{16}s_{35}} \\ &+ \frac{\langle 2\,3\rangle\langle 2\,6\rangle[4\,1]\langle 5|(1+4)|6\rangle}{[2\,3]\langle 1\,2\rangle\langle 4\,6\rangle s_{16}s_{35}} + \frac{\langle 2\,6\rangle\langle 1|(2+6)|3\rangle^2\langle 4|(3+5)|2\rangle}{\langle 5\,4\rangle s_{12}s_{16}s_{35}t_{126}} \end{aligned} \tag{3.2}$$

$$
\begin{aligned}
-iA_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^+, g_3^-, \gamma_4^+) &= -\frac{\langle 1\,3\rangle^2\langle 2|(1+3)|6\rangle^2}{\langle 5\,4\rangle\langle 4\,6\rangle s_{12}s_{23}t_{123}} + \frac{[2\,6][4\,5]\langle 6\,1\rangle\langle 5\,3\rangle\langle 2|(4+5)|3\rangle}{[1\,6]\langle 5\,4\rangle s_{12}s_{35}t_{126}} \\
&+ \frac{[2\,5]^2\langle 6\,1\rangle\langle 4|(2+5)|3\rangle}{[1\,6][5\,3]\langle 4\,6\rangle s_{23}t_{146}} + \frac{\langle 1\,6\rangle[2\,5]\langle 2|(1+3)|5\rangle\langle 2|(1+6)|3\rangle}{[1\,6][2\,3]\langle 5\,4\rangle\langle 4\,6\rangle s_{12}s_{35}} \\
&+ \frac{\langle 1\,3\rangle\langle 2|(1+3)|6\rangle\langle 2|(1+6)|3\rangle}{[1\,6]\langle 5\,4\rangle\langle 4\,6\rangle s_{12}s_{23}} + \frac{\langle 1\,6\rangle[2\,5]\langle 1\,3\rangle\langle 4|(2+5)|3\rangle}{[1\,6]\langle 4\,6\rangle\langle 2\,1\rangle s_{23}s_{35}}
\end{aligned}
\tag{3.3}
$$

These amplitudes have been computed directly from the color-ordered Feynman diagrams. Equivalent but less compact expressions could be obtained by summing over permutations of the two-quark four-gluon amplitudes,

$$
\begin{aligned}
A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_2, g_3, \gamma_4) = A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_2, g_3, g_4) + A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_2, g_4, g_3) \\
A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_4, g_2, g_3) + A_6^{\rm tree}(\bar{q}_5, q_6, g_4, g_1, g_2, g_3)\,.
\end{aligned}
\tag{3.4}
$$

An alternative method is to use supersymmetric Ward identities to obtain the result from the known two-quark four-gluon and six-gluon amplitudes (see Appendix A).

3.2 Four-quark, one-gluon, one-photon amplitudes

The color decomposition of the four-quark, one-gluon, one-photon amplitudes is given in eq. (2.14), with $m = r = 1$. For each helicity configuration there are four independent color structures. For the configurations of type $(-,-,+,+,+,+)$, the amplitudes are given in eq. (2.15), with $m = 1$ and $(k,l) = (1,0)$ and $(0,1)$.

Table 2 lists the two distinct helicity configurations of type $(-,-,-,+,+,+)$. The

	$\bar{q}_1$	q_2	$\overline{Q}_3$	Q_4	g_5	γ_6
I	$+$	$-$	$+$	$-$	$-$	$+$
II	$+$	$-$	$-$	$+$	$-$	$+$

Table 2: The distinct helicity configurations of type $(-,-,-,+,+,+)$, for the four-quark, one-gluon, one-photon amplitudes.

color-ordered sub-amplitudes for these configurations are

$$
\begin{aligned}
A_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^+, Q_4^-; \emptyset; g_5^-; \gamma_6^+) &= Q_q f_1(1,2,3,4;5;6) + Q_Q f_2(3,4,1,2;5;6) \\
A_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^+, Q_4^-; g_5^-; \emptyset; \gamma_6^+) &= Q_q f_2(1,2,3,4;5;6) + Q_Q f_1(3,4,1,2;5;6) \\
B_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^+, Q_4^-; g_5^-; \emptyset; \gamma_6^+) &= Q_q g_1(1,2,3,4;5;6) + Q_Q g_2(3,4,1,2;5;6) \\
B_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^+, Q_4^-; \emptyset; g_5^-; \gamma_6^+) &= Q_q g_2(1,2,3,4;5;6) + Q_Q g_1(3,4,1,2;5;6)
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
A_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^-, Q_4^+; \emptyset; g_5^-; \gamma_6^+) &= \quad Q_q f_3(1,2,3,4;5;6) + Q_Q f_3(4,3,2,1;5;6) \\
A_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^-, Q_4^+; g_5^-; \emptyset; \gamma_6^+) &= \quad Q_q f_4(1,2,3,4;5;6) + Q_Q f_4(4,3,2,1;5;6) \\
B_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^-, Q_4^+; g_5^-; \emptyset; \gamma_6^+) &= \quad Q_q g_1(1,2,4,3;5;6) - Q_Q g_2(4,3,1,2;5;6) \\
B_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^-, Q_4^+; \emptyset; g_5^-; \gamma_6^+) &= -Q_q g_2(1,2,4,3;5;6) + Q_Q g_1(4,3,1,2;5;6)
\end{aligned}
\tag{3.6}
$$

where

$$
\begin{aligned}
-if_1(1,2,3,4;5;6) &= \frac{\langle 3|(1+6)|2\rangle\langle 3|(1+6)|4\rangle}{\langle 1\,6\rangle\langle 6\,2\rangle[3\,5][5\,2]s_{34}} \\
&\quad + \frac{\langle 5\,2\rangle[3\,1]\langle 6|(2+5)|4\rangle}{[5\,2]\langle 6\,2\rangle s_{34}t_{134}} + \frac{\langle 4\,5\rangle\langle 3|(1+6)|2\rangle^2}{\langle 1\,6\rangle\langle 6\,2\rangle[3\,5]s_{34}t_{126}}
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
-if_2(1,2,3,4;5;6) &= \frac{\langle 1|(4+5)|2\rangle\langle 3|(1+6)|2\rangle}{\langle 1\,6\rangle\langle 6\,2\rangle[1\,5][5\,4]s_{34}} \\
&\quad + \frac{[1\,6]\langle 2\,4\rangle\langle 3|(1+6)|5\rangle}{\langle 1\,6\rangle[1\,5]s_{34}t_{156}} + \frac{\langle 5\,3\rangle\langle 3|(1+6)|2\rangle^2}{\langle 1\,6\rangle\langle 6\,2\rangle[5\,4]s_{34}t_{126}}
\end{aligned}
\tag{3.8}
$$

$$
\begin{aligned}
-ig_1(1,2,3,4;5;6) &= \frac{\langle 1|(3+6)|4\rangle\langle 3|(1+6)|2\rangle}{\langle 1\,6\rangle\langle 6\,2\rangle[1\,5][5\,2]s_{34}} \\
&\quad + \frac{[1\,6]\langle 2\,4\rangle\langle 3|(1+6)|5\rangle}{\langle 1\,6\rangle[1\,5]s_{34}t_{156}} + \frac{\langle 5\,2\rangle[3\,1]\langle 6|(2+5)|4\rangle}{[5\,2]\langle 6\,2\rangle s_{34}t_{134}}
\end{aligned}
\tag{3.9}
$$

$$
-ig_2(1,2,3,4;5;6) = -\frac{\langle 3|(1+6)|2\rangle^2}{\langle 1\,6\rangle\langle 6\,2\rangle[3\,5][5\,4]t_{126}}
\tag{3.10}
$$

$$
\begin{aligned}
-if_3(1,2,3,4;5;6) &= -\frac{t_{146}\langle 4|(1+6)|2\rangle}{\langle 1\,6\rangle\langle 6\,2\rangle[3\,5][5\,2]s_{34}} \\
&\quad + \frac{\langle 5\,4\rangle\langle 4|(1+6)|2\rangle^2}{\langle 1\,6\rangle\langle 6\,2\rangle[3\,5]s_{34}t_{126}} + \frac{\langle 5\,2\rangle[4\,1]\langle 6|(2+5)|3\rangle}{[5\,2]\langle 6\,2\rangle s_{34}t_{134}}
\end{aligned}
\tag{3.11}
$$

$$
\begin{aligned}
-if_4(1,2,3,4;5;6) &= \frac{[4\,1]\langle 2\,3\rangle\langle 4|(1+6)|2\rangle}{\langle 1\,6\rangle\langle 6\,2\rangle[1\,5][5\,4]s_{34}} \\
&\quad + \frac{[1\,6]\langle 2\,3\rangle\langle 4|(1+6)|5\rangle}{\langle 1\,6\rangle[1\,5]s_{34}t_{156}} + \frac{\langle 3\,5\rangle\langle 4|(1+6)|2\rangle^2}{\langle 1\,6\rangle\langle 6\,2\rangle[5\,4]s_{34}t_{126}}
\end{aligned}
\tag{3.12}
$$

All other configurations are related to these by parity inversion and charge conjugation[3]. In performing charge conjugation, one must exchange each quark with its anti-quark ($\bar{q}_i \leftrightarrow q_i$ for $i = 1, 2$). For example, by applying charge conjugation and parity inversion on eq.(3.5), we obtain

$$
A_6^{\rm tree}(\bar{q}_1^-, q_2^+; \overline{Q}_3^-, Q_4^+; \emptyset; g_5^-; \gamma_6^+) = Q_q f_2(2,1,4,3;5;6) + Q_Q f_1(4,3,2,1;5;6)
$$

[3] When photons are present in the sub-amplitudes, we apply charge conjugation on the kinematic part only, without flipping the sign of the quark electric charge.

$$\begin{aligned}
A_6^{\rm tree}(\bar{q}_1^-, q_2^+; \overline{Q}_3^-, Q_4^+; g_5^-; \emptyset; \gamma_6^+) &= Q_q f_1(2,1,4,3;5;6) + Q_Q f_2(4,3,2,1;5;6) \\
B_6^{\rm tree}(\bar{q}_1^-, q_2^+; \overline{Q}_3^-, Q_4^+; g_5^-; \emptyset; \gamma_6^+) &= Q_q g_1(2,1,4,3;5;6) + Q_Q g_2(4,3,2,1;5;6) \\
B_6^{\rm tree}(\bar{q}_1^-, q_2^+; \overline{Q}_3^-, Q_4^+; \emptyset; g_5^-; \gamma_6^+) &= Q_q g_2(2,1,4,3;5;6) + Q_Q g_1(4,3,2,1;5;6)\,.
\end{aligned} \qquad (3.13)$$

4 Two-Photon Amplitudes

The amplitudes with two photons can be used for NLO two-photons + one-jet production at hadron colliders, for NLO photon + two-jet production in direct photo-production at electron-proton colliders, or for NLO three-jet production in gamma-gamma scattering at some future e^+e^- collider. They also contribute to NNLO calculations of the same processes with one less jet in the final state.

These amplitudes are of particular importance to the evaluation of the irreducible backgrounds to low-mass Higgs searches at LHC; they contribute to the NLO evaluation of the background to Higgs + one-jet production and to the NNLO evaluation of the background to inclusive Higgs production.

The tree-level two-photon amplitudes with six external legs are

a) two-quark, two-gluon, two-photon amplitudes;

b) four-quark, two-photon amplitudes.

4.1 Two-quark, two-gluon, two-photon amplitudes

The color decomposition of the two-quark, two-gluon, two-photon amplitudes is given in eq. (2.7). For the helicity configurations of type $(-,-,+,+,+,+)$, the sub-amplitudes are given in eq. (2.9), with $r = m = 2$. There are three distinct configurations of type $(-,-,-,+,+,+)$, which are listed in table 3. The other helicity configurations are obtained from these by particle relabeling, parity inversion and/or charge conjugation.

	$\bar{q}_5$	q_6	g_1	g_2	γ_3	γ_4
I	$+$	$-$	$-$	$-$	$+$	$+$
II	$+$	$-$	$+$	$-$	$-$	$+$
III	$+$	$-$	$-$	$+$	$-$	$+$

Table 3: The distinct helicity configurations of type $(-,-,-,+,+,+)$, for the two-quark, two-gluon, two-photon amplitudes.

$$-\,iA_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^-, \gamma_3^+, \gamma_4^+) \;=\; \frac{\langle 5|(1+2)|6\rangle\langle 5\,6\rangle t_{126}}{[5\,2][1\,6][2\,1]\langle 6\,3\rangle\langle 6\,4\rangle\langle 5\,3\rangle\langle 5\,4\rangle}$$

$$- \frac{\langle 1\,6\rangle[4\,5]\langle 3|(1+6)|2\rangle}{[1\,6]\langle 4\,5\rangle[5\,2]\langle 6\,3\rangle t_{136}} - \frac{\langle 1\,6\rangle[3\,5]\langle 4|(1+6)|2\rangle}{[1\,6]\langle 3\,5\rangle[5\,2]\langle 6\,4\rangle t_{146}} \tag{4.1}$$

$$\begin{aligned}
-iA_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^+, g_2^-, \gamma_3^-, \gamma_4^+) &= \frac{[4\,5]\langle 2\,6\rangle^2\langle 1|(2+6)|3\rangle}{\langle 4\,5\rangle[5\,3]\langle 1\,6\rangle s_{12}t_{126}} + \frac{\langle 3\,6\rangle[5\,4]\langle 1|(3+6)|2\rangle}{[3\,6]\langle 5\,4\rangle[5\,2]\langle 1\,6\rangle t_{136}} \\
&+ \frac{\langle 3\,6\rangle[1\,5]^2\langle 4|(3+6)|2\rangle}{[3\,6][2\,5]\langle 6\,4\rangle s_{12}t_{125}} + \frac{[5\,1]\langle 6\,2\rangle\langle 5|(1+4)|6\rangle t_{145}}{[3\,6][5\,2][5\,3]\langle 6\,1\rangle\langle 6\,4\rangle\langle 4\,5\rangle s_{12}} \\
&+ \frac{\langle 1|(3+6)|2\rangle\left([5\,1]\langle 1\,6\rangle\langle 5|(2+3)|6\rangle + [5\,4]\langle 4\,6\rangle[5\,2]\langle 2\,6\rangle\right)}{[3\,6][5\,2][5\,3]\langle 6\,1\rangle\langle 6\,4\rangle\langle 4\,5\rangle s_{12}}
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
-iA_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^+, \gamma_3^-, \gamma_4^+) &= \frac{\langle 1\,6\rangle[2\,5]\langle 4|(2+5)|3\rangle}{[1\,6]\langle 2\,5\rangle[5\,3]\langle 6\,4\rangle t_{146}} + \frac{\langle 1\,6\rangle[4\,5][6\,2]\langle 2|(4+5)|3\rangle}{[1\,6]\langle 4\,5\rangle[5\,3]s_{12}t_{126}} \\
&+ \frac{\langle 3\,6\rangle[2\,5]\langle 5\,1\rangle\langle 4|(2+5)|1\rangle}{[3\,6]\langle 2\,5\rangle\langle 6\,4\rangle s_{12}t_{346}} + \frac{\langle 1\,6\rangle[2\,5]\langle 2|(4+5)|3\rangle}{[1\,6][3\,5]\langle 5\,4\rangle\langle 4\,6\rangle s_{12}} \\
&+ \frac{t_{136}\left(\langle 5\,1\rangle[2\,6][5\,3]\langle 3\,6\rangle + [1\,2][6\,5]\langle 1\,6\rangle\langle 5\,1\rangle\right)}{[1\,6][5\,3][3\,6]\langle 2\,5\rangle\langle 5\,4\rangle\langle 4\,6\rangle s_{12}}
\end{aligned} \tag{4.3}$$

These expressions were obtained by direct computation of the color-ordered Feynman diagrams. Equivalent representations can easily be obtained by expressing the amplitudes as linear combinations of the one photon amplitudes,

$$\begin{aligned}
A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_2, \gamma_3, \gamma_4) = A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_2, g_3, \gamma_4) + \\
A_6^{\rm tree}(\bar{q}_5, q_6, g_3, g_1, g_2, \gamma_4) + A_6^{\rm tree}(\bar{q}_5, q_6, g_1, g_3, g_2, \gamma_4)\,.
\end{aligned} \tag{4.4}$$

Alternatively, they can be obtained from two-quark four-gluon and six-gluon amplitudes using supersymmetric Ward identities (see Appendix B).

4.2 Four-quark, two-photon amplitudes

The color decomposition of the four-quark, two-photon amplitudes is given in eq. (2.14). For the helicity configurations of type $(-,-,+,+,+,+)$, the sub-amplitudes are given in eq. (2.15), with $m=2$ and $r=0$. The color strings reduce to Kronecker delta's, so that the amplitude is written as a single kinematic term multiplying the color factor

$$\delta_{i_4}^{\bar{\imath}_1}\delta_{i_2}^{\bar{\imath}_3} - \frac{1}{N}\delta_{i_2}^{\bar{\imath}_1}\delta_{i_4}^{\bar{\imath}_3}.$$

There are two distinct helicity configurations of type $(-,-,-,+,+,+)$ which are listed in table 4. All other configurations are related to these by parity inversion and charge conjugation. The amplitudes can be written in terms of two functions g_1 and g_2, which are identical to the $g_{1,2}$ of eqs. (3.9) - (3.10),

$$A_6^{\rm tree}(\bar{q}_1^+, q_2^-; \overline{Q}_3^+, Q_4^-; \gamma_5^-; \gamma_6^+) \;=\; Q_q^2 g_1(1,2,3,4;5,6) + Q_Q^2 g_1(3,4,1,2;5,6)$$

	$\bar{q}_1$	q_2	$\overline{Q}_3$	Q_4	γ_5	γ_6
I	+	−	+	−	−	+
II	+	−	−	+	−	+

Table 4: The distinct helicity configurations of type $(-,-,-,+,+,+)$, for the four-quark, two-photon amplitudes.

$$
\begin{aligned}
& \quad + \quad Q_q Q_Q (g_2(1,2,3,4;5,6) + g_2(3,4,1,2;5,6)) \\
A_6^{\text{tree}}(\bar{q}_1^+, q_2^-; \overline{Q}_3^-, Q_4^+; \gamma_5^-; \gamma_6^+) &= Q_q^2 g_1(1,2,4,3;5,6) + Q_Q^2 g_1(4,3,1,2;5,6) \\
& \quad - \quad Q_q Q_Q (g_2(1,2,4,3;5,6) + g_2(4,3,1,2;5,6))\,.
\end{aligned}
\tag{4.5}
$$

5 Three-Photon Amplitudes

The amplitudes with three photons can be used for NLO three-photon production at hadron colliders, for NLO two-photon inclusive production and two-photon + one-jet production in direct photo-production at electron-proton colliders, or for NLO photon + one- or two-jet inclusive production in gamma-gamma collisions.

For NLO calculations we need two-quark, one-gluon, three-photon tree-level amplitudes. For the helicity configurations of type $(-,-,+,+,+,+)$, the sub-amplitudes are given in eq. (2.9), with $r = 1$ and $m = 3$. There is one distinct helicity configuration of type $(-,-,-,+,+,+)$, for which the sub-amplitude is

$$
\begin{aligned}
& -iA_6^{\text{tree}}(\bar{q}_5^+, q_6^-, g_1^-, \gamma_2^-, \gamma_3^+, \gamma_4^+) = \\
& \qquad \frac{s_{56} t_{612} \langle 5|(3+4)|6\rangle}{\langle 5\,3\rangle\langle 3\,6\rangle\,\langle 5\,4\rangle\langle 4\,6\rangle\,[5\,1][1\,6]\,[5\,2][2\,6]} + \sum_{\sigma\in S_2,\rho\in S_2} \frac{[5\,\rho_3]\langle 6\,\sigma_2\rangle\langle \rho_4|5|\rho_3\rangle\sigma_1}{\langle 5\,\rho_3\rangle[6\,\sigma_2]\langle 6\,\rho_4\rangle[5\,\sigma_1]t_{6\rho_4\sigma_2}}\,.
\end{aligned}
\tag{5.1}
$$

We note that this is also the sub-amplitude for the two-quark, four-photon process, since only the color factor differs. Indeed, this sub-amplitude is also identical to that obtained for $e^+e^- \to \gamma\gamma\gamma\gamma$ [17].

6 One-Loop Amplitudes

The results in this section are obtained either in their entirety or from discussions contained in references [15, 16]. Our purpose here is to present all of the results in terms of known components so that they can be combined with the tree-level results of the previous sections to produce complete NLO calculations. When supplemented with higher-order terms in ϵ, they also contribute to NNLO calculations.

The color decomposition of one-loop amplitudes in QCD is somewhat more complicated than it is for tree-level amplitudes. If there are external gluons, one finds new color structures at one-loop. Associated with each color structure is a "partial amplitude"[15]. For instance, the two-quark, three-gluon one-loop matrix element is written as

$$\begin{aligned}
\mathcal{A}^{\text{1-loop}}(\bar{q}_1, q_2; g_3, g_4, g_5) \;=\; g^5 \Bigg\{ & \sum_{\sigma\in S_3} N_c \, (T^{\sigma_3}T^{\sigma_4}T^{\sigma_5})_{i_2}^{\;\bar{\imath}_1} \, A_{5;1}(\bar{q}_1, q_2; g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) \\
+ & \sum_{\sigma\in S_3} \mathrm{Tr}\,(T^{\sigma_3})\,(T^{\sigma_4}T^{\sigma_5})_{i_2}^{\;\bar{\imath}_1} \, A_{5;2}(\bar{q}_1, q_2; g_{\sigma_3}; g_{\sigma_4}, g_{\sigma_5}) \\
+ & \sum_{\sigma\in S_3/Z_2} \mathrm{Tr}\,(T^{\sigma_3}T^{\sigma_4})\,(T^{\sigma_5})_{i_2}^{\;\bar{\imath}_1} \, A_{5;3}(\bar{q}_1, q_2; g_{\sigma_3}, g_{\sigma_4}; g_{\sigma_5}) \\
+ & \sum_{\sigma\in S_3/Z_3} \mathrm{Tr}\,(T^{\sigma_3}T^{\sigma_4}T^{\sigma_5})\, \delta_{i_2}^{\bar{\imath}_1} A_{5;4}(\bar{q}_1, q_2; g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) \Bigg\}
\end{aligned} \tag{6.1}$$

where the sums exclude permutations that leave the color factors invariant. For instance, in the third term the trace is unchanged if elements 3 and 4 are interchanged. Thus σ takes values in the set

$$S_3/Z_2 = \{(345), (354), (453)\}. \tag{6.2}$$

Note that the color factor associated with the $A_{5;2}$ term vanishes, since $\mathrm{Tr}(T^a) = 0$ when T^a is a generator of $SU(N_c)$. For this reason, $A_{5;2}$ terms are normally dropped. In the amplitudes below, we keep the $A_{5;2}$ terms so that one can follow all of the terms as gluons are converted into photons.

6.1 Two-quark amplitudes

The partial amplitudes are most efficiently expressed in terms of a set of gauge-invariant, color-ordered building blocks called "primitive amplitudes" [15]. For the two-quark amplitudes, it is convenient to construct two primitive amplitudes for each internal spin state (*i.e.* spin of the particle going around the loop), $A_n^{L,[J]}(\bar{q}_1, g_3, \ldots, q_2, \ldots, g_n)$ and $A_n^{R,[J]}(\bar{q}_1, g_3, \ldots, q_2, \ldots, g_n)$, $J = 1, \frac{1}{2}, 0$, with the L and R primitive amplitudes related by reversing the cyclic ordering of the external legs:

$$A_n^{R,[J]}(\bar{q}_1, g_3, \ldots, q_2, \ldots, g_n) = (-1)^n A_n^{L,[J]}(\bar{q}_1, g_n, \ldots, q_2, \ldots, g_3). \tag{6.3}$$

Spin 1, of course, refers to gluons in the loop, while spins $\frac{1}{2}$ and 0 refer to massless $(N_c + \overline{N}_c)$ representation Weyl fermions, and complex scalars respectively.

It turns out that the R-type primitives are not needed for $J = \frac{1}{2}, 0$, so we abbreviate the notation as

$$A_n^{L,[1]} \to A_n^L \qquad A_n^{R,[1]} \to A_n^R \qquad A_n^{L,[\frac{1}{2}]} \to A_n^{[\frac{1}{2}]} \qquad A_n^{L,[0]} \to A_n^{[0]}. \tag{6.4}$$

The $A_{n;1}$ partial amplitudes have the simplest representation in terms of primitive amplitudes:

$$\begin{aligned} A_{n;1}(\bar{q}_1, q_2, g_3, \ldots, g_n) &= A_n^L(\bar{q}_1, q_2, g_3, \ldots, g_n) - \frac{1}{N_c^2} A_n^R(\bar{q}_1, q_2, g_3, \ldots, g_n) \\ &\quad + \frac{n_f}{N_c} A_n^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_3, \ldots, g_n) + \frac{n_s}{N_c} A_n^{[0]}(\bar{q}_1, q_2, g_3, \ldots, g_n), \end{aligned} \tag{6.5}$$

where n_f is the number of fermions and n_s is the number of scalars. In these partial amplitudes, only those primitive amplitudes with the same ordering of legs as the color factor contribute.

The partial amplitudes $A_{n;c>1}$ are expressed as sums over primitive amplitudes with various permutations of the external legs. For the two-quark amplitudes [15],

$$\begin{aligned} & A_{n;c}(\bar{q}_1, q_2; g_3, \ldots, g_{c+1}; g_{c+2}, \ldots, g_n) \\ & \qquad = (-1)^{c-1} \sum_{\sigma \in \mathrm{COP}\{\alpha\}\{\beta\}} \left[A_n^L(\sigma) - \frac{n_f}{N_c} A_n^{R,[\frac{1}{2}]}(\sigma) - \frac{n_s}{N_c} A_n^{R,[0]}(\sigma) \right], \end{aligned} \tag{6.6}$$

where $\{\alpha\} \equiv \{g_{c+1}, g_c, \ldots, g_3\}$ and $\{\beta\} \equiv \{\bar{q}_1, q_2, g_{c+2}, g_{c+3}, \ldots, g_n\}$, and $\mathrm{COP}\{\alpha\}\{\beta\}$ is the set of permutations of $\{\bar{q}_1, q_2, \ldots, g_n\}$, with the position of $\bar{q}_1$ held fixed that preserve the cyclic ordering of the α_i within $\{\alpha\}$ and of the β_j within $\{\beta\}$ while allowing for all possible relative orderings of the α_i with respect to the β_j. Expanding this expression, and making use of symmetry relations involving $A^{[\frac{1}{2},0]}$ [15], yields

$$\begin{aligned} A_{5;2}(\bar{q}_1, q_2; g_3; g_4, g_5) &= -A_5^L(\bar{q}_1, q_2, g_4, g_5, g_3) - A_5^L(\bar{q}_1, q_2, g_4, g_3, g_5) \\ &\quad -A_5^L(\bar{q}_1, q_2, g_3, g_4, g_5) - A_5^L(\bar{q}_1, g_3, q_2, g_4, g_5) \end{aligned} \tag{6.7}$$

$$\begin{aligned} A_{5;3}(\bar{q}_1, q_2; g_4, g_5; g_3) &= \sum_{\sigma \in S_3} A_5^L(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) - \sum_{\sigma \in Z_2} A_5^R(\bar{q}_1, g_3, q_2, g_{\sigma_4}, g_{\sigma_5}) \\ &\quad + \sum_{\sigma \in Z_2} A_5^L(\bar{q}_1, g_4, q_2, g_{\sigma_3}, g_{\sigma_5}) + \sum_{\sigma \in Z_2} A_5^L(\bar{q}_1, g_5, q_2, g_{\sigma_3}, g_{\sigma_4}) \end{aligned} \tag{6.8}$$

$$\begin{aligned} A_{5;4}(\bar{q}_1, q_2; g_3, g_4, g_5) &= \sum_{\sigma \in Z_3} \Big\{ A_5^R(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) + A_5^R(\bar{q}_1, g_{\sigma_3}, q_2, g_{\sigma_4}, g_{\sigma_5}) \\ &\quad - A_5^L(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_5}, g_{\sigma_4}) - A_5^L(\bar{q}_1, g_{\sigma_3}, q_2, g_{\sigma_5}, g_{\sigma_4}) \\ &\quad - \frac{n_f}{N_c} \left[A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) + A_5^{[\frac{1}{2}]}(\bar{q}_1, g_{\sigma_3}, q_2, g_{\sigma_4}, g_{\sigma_5}) \right] \\ &\quad - \frac{n_s}{N_c} \left[A_5^{[0]}(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) + A_5^{[0]}(\bar{q}_1, g_{\sigma_3}, q_2, g_{\sigma_4}, g_{\sigma_5}) \right] \Big\}. \end{aligned} \tag{6.9}$$

The explicit expressions for the primitive amplitudes are given in reference [15]. For reasons of computational simplicity, the results presented there are in terms of a different

linear combination of primitive amplitudes, labeled A^L, $A^{\rm SUSY}$, A^f and A^s, which are related to those used here by

$$\begin{aligned}
A^L(\sigma) &= A^L(\sigma) \\
A^R(\sigma) &= A^{\rm SUSY}(\sigma) - A^L(\sigma) + A^f(\sigma) + A^s(\sigma) \\
A^{[\frac{1}{2}]}(\sigma) &= -A^f(\sigma) - A^s(\sigma) \\
A^{[0]}(\sigma) &= A^s(\sigma)\,.
\end{aligned} \tag{6.10}$$

In all generality the expression for $A^R(\sigma)$ should have a $-A^{R,[\frac{1}{2}]}(\sigma)$ term on the right-hand side. For all orderings that appear below this term vanishes.

6.2 One-loop corrections to $\bar{q}qgg\gamma$ amplitudes

This process has been discussed thoroughly by Signer [16]. We present the results directly in terms of the $\bar{q}qggg$ primitive amplitudes of Bern, Dixon and Kosower [15]. In order to convert gluons into photons, we follow the prescription in reference [15][4]. This is essentially the same procedure outlined in section 2.2, but for a few subtleties that arise at one loop. One such subtlety is to note that when there are fermion loops, the photon can couple to either the external quark line (external coupling) or to the loop fermion (internal coupling). For external coupling, one finds a factor of $Q_q n_f$, the charge of the external quark times the number of fermion flavors contributing loops. For internal coupling, one finds a factor of $\sum_{i=1}^{n_f} Q_{q_i}$, the sum of charges of the different fermion flavors appearing in the loop, which we abbreviate as $\mathrm{Tr}\,[Q_f]$. A second subtlety is related and involves a term which contributes to $A_{5;1}(\bar{q}_1, q_2, g_3, g_4, \gamma_5)$, but vanishes in $A_{5;1}(\bar{q}_1, q_2, g_3, g_4, g_5)$,

$$\delta A_{5;1}(\bar{q}_1, q_2, g_3, g_4, \gamma_5) = \frac{1}{N_c}\left(\mathrm{Tr}\,[Q_f] - Q_q n_f\right) \sum_{\sigma\in \mathrm{COP}\{g_5\}\{\bar{q}_1 q_2 g_3 g_4\}} A_5^{[\frac{1}{2}]}(\sigma). \tag{6.11}$$

This term vanishes in pure QCD because all quarks carry the same color charge. Because they do not carry the same electric charge, the term reappears when a gluon is changed into a photon.

In the results below, we drop all reference to any hypothetical scalar particles. They can be added back in by noting that scalar terms enter in exactly the same way as the fermion terms do. Thus, to reinsert the scalars, simply duplicate the $A^{[\frac{1}{2}]}$ terms and replace $n_f \to n_s$, $\mathrm{Tr}\,[Q_f] \to \mathrm{Tr}\,[Q_s]$ and $A^{[\frac{1}{2}]} \to A^{[0]}$.

The full one-loop $\overline{q}qgg\gamma$ amplitude is

$$\mathcal{A}^{\text{1-loop}}(\bar{q}_1, q_2; g_3, g_4; \gamma_5) \;=\; \sqrt{2}eg^4 \left\{ \sum_{\sigma\in Z_2} N_c \left(T^{\sigma_3}T^{\sigma_4}\right)_{i_2}^{\bar{\imath}_1} A_{5;1}(\bar{q}_1, q_2; g_{\sigma_3}, g_{\sigma_4}; \gamma_5) \right.$$

[4] Note that on this topic, the e-print supersedes the journal reference.

$$
+ \sum_{\sigma \in Z_2} \mathrm{Tr}\,(T^{\sigma_3})\,(T^{\sigma_4})_{i_2}^{\bar{\imath}_1}\, A_{5;2}(\bar{q}_1, q_2; g_{\sigma_3}; g_{\sigma_4}; \gamma_5)
+ \mathrm{Tr}\left(T^3 T^4\right) \delta_{i_2}^{\bar{\imath}_1} A_{5;3}(\bar{q}_1, q_2; g_3, g_4; \gamma_5)\Big\}\,, \tag{6.12}
$$

where the partial amplitudes are given by

$$
\begin{aligned}
A_{5;1}(\bar{q}_1, q_2; g_3, g_4; \gamma_5) \;=\;& -Q_q \Bigg\{ A_5^L(\bar{q}_1, g_5, q_2, g_3, g_4) + \frac{n_f}{N_c} A_5^{[\frac{1}{2}]}(\bar{q}_1, g_5, q_2, g_3, g_4) \\
& + \frac{1}{N_c^2} \Big[A_5^R(\bar{q}_1, q_2, g_3, g_4, g_5) + A_5^R(\bar{q}_1, q_2, g_3, g_5, g_4) \\
& \qquad + A_5^R(\bar{q}_1, q_2, g_5, g_3, g_4) \Big] \Bigg\} \\
& + \frac{\mathrm{Tr}\,[Q_f]}{N_c} \Big\{ A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_3, g_4, g_5) + A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_3, g_5, g_4) \\
& \qquad + A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_5, g_3, g_4) + A_5^{[\frac{1}{2}]}(\bar{q}_1, g_5, q_2, g_3, g_4) \Big\}
\end{aligned} \tag{6.13}
$$

$$
A_{5;2}(\bar{q}_1, q_2; g_3; g_4; \gamma_5) \;=\; Q_q \Bigg\{ \sum_{\sigma \in Z_2} A_5^L(\bar{q}_1, g_5, q_2, g_{\sigma_3}, g_{\sigma_4}) - \sum_{\sigma \in Z_2} A_5^R(\bar{q}_1, g_4, q_2, g_{\sigma_3}, g_{\sigma_5}) \Bigg\} \tag{6.14}
$$

$$
\begin{aligned}
A_{5;3}(\bar{q}_1, q_2; g_3, g_4; \gamma_5) \;=\;& -Q_q \sum_{\sigma \in Z_2} \Big\{ A_5^L(\bar{q}_1, g_5, q_2, g_{\sigma_3}, g_{\sigma_4}) + A_5^R(\bar{q}_1, g_5, q_2, g_{\sigma_3}, g_{\sigma_4}) \Big\} \\
& + \sum_{\sigma \in S_3} \Bigg\{ Q_q A_5^R(\bar{q}_1, g_{\sigma_3}, q_2, g_{\sigma_4}, g_{\sigma_5}) + Q_q A_5^R(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) \\
& \qquad - \frac{\mathrm{Tr}\,[Q_f]}{N_c} A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) \Bigg\}.
\end{aligned} \tag{6.15}
$$

The one-loop correction to $\sigma_{\bar{q}qgg\gamma}$ is

$$
\sigma_{\bar{q}qgg\gamma}^{\text{1-loop}} \;=\; 4e^2 g^6 (N_c^2 - 1) \sum_{\text{helicities}} \mathrm{Re} \sum_{\sigma \in Z_2} A_5^{\text{tree}*}(\bar{q}_1, q_2; g_{\sigma_3}, g_{\sigma_4}; \gamma_5) \times \tag{6.16}
$$

$$
\Big\{ (N_c^2 - 1) A_{5;1}(\bar{q}_1, q_2; g_{\sigma_3}, g_{\sigma_4}; \gamma_5) - A_{5;1}(\bar{q}_1, q_2; g_{\sigma_4}, g_{\sigma_3}; \gamma_5) + A_{5;3}(\bar{q}_1, q_2; g_3, g_4; \gamma_5) \Big\}.
$$

6.3 One-loop corrections to $\bar{q}qg\gamma\gamma$ amplitudes

This process has also been discussed by Signer [16], but in much less detail than the $\bar{q}qgg\gamma$ process. By breaking the permutation sum up into primitive components we are able to obtain far more manageable (though still large) expressions. The color decomposition for this process is

$$
\begin{aligned}
\mathcal{A}^{\text{1-loop}}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5) \;=\;& 2e^2 g^3 \Big\{ N_c \left(T^3\right)_{i_2}^{\bar{\imath}_1} A_{5;1}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5) \\
& + \mathrm{Tr}\left(T^3\right) \delta_{i_2}^{\bar{\imath}_1} A_{5;2}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5) \Big\}\,.
\end{aligned} \tag{6.17}
$$

The primitive decompositions of the partial amplitudes are

$$\begin{aligned}
A_{5;1}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5) &= -Q_q^2 \sum_{\sigma\in Z_2} A_5^R(\bar{q}_1, g_3, q_2, g_{\sigma_4}, g_{\sigma_5}) \\
&\quad - \sum_{\sigma\in S_3} \left[\frac{Q_q^2}{N_c^2} A_5^R(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) - \frac{\mathrm{Tr}\left[Q_f^2\right]}{N_c} A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) \right] \quad (6.18) \\
A_{5;2}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5) &= Q_q^2 \sum_{\sigma\in Z_2} A_5^R(\bar{q}_1, g_3, q_2, g_{\sigma_4}, g_{\sigma_5}) \\
&\quad + \sum_{\sigma\in S_3} \left[Q_q^2 A_5^R(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) - \frac{\mathrm{Tr}\left[Q_f^2\right]}{N_c} A_5^{[\frac{1}{2}]}(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}) \right], \quad (6.19)
\end{aligned}$$

where $\mathrm{Tr}\left[Q_f^2\right]$ is the sum of the squared charges of all n_f quarks. The one-loop correction to the cross section is

$$\sigma^{\text{1-loop}}_{\bar{q}qg\gamma\gamma} = 8e^4 g^4 N_c(N_c^2-1) \sum_{\text{helicities}} \mathrm{Re}\left[A_5^{\text{tree}*}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5) A_{5;1}(\bar{q}_1, q_2; g_3; \gamma_4, \gamma_5)\right]. \quad (6.20)$$

6.4 One-loop corrections to $\bar{q}q\gamma\gamma\gamma$ amplitudes

This process has not been discussed in detail before. The color structure is trivial,

$$\mathcal{A}^{\text{1-loop}}(\bar{q}_1, q_2; \gamma_3, \gamma_4, \gamma_5) = 2\sqrt{2} e^3 g^2 N_c \delta_{i_2}^{\bar{\imath}_1} A_{5;1}(\bar{q}_1, q_2; \gamma_3, \gamma_4, \gamma_5), \quad (6.21)$$

with

$$A_{5;1}(\bar{q}_1, q_2; \gamma_3, \gamma_4, \gamma_5) = Q_q^3 \left(1 - \frac{1}{N_c^2}\right) \sum_{\sigma\in S_3} A_5^R(\bar{q}_1, q_2, g_{\sigma_3}, g_{\sigma_4}, g_{\sigma_5}). \quad (6.22)$$

The one-loop correction to the cross section is

$$\sigma^{\text{1-loop}}_{\bar{q}q\gamma\gamma\gamma} = 16 e^6 g^2 Q_q^6 N_c^2 \sum_{\text{helicities}} \mathrm{Re}\left[A_5^{\text{tree}*}(\bar{q}_1, q_2; \gamma_3, \gamma_4, \gamma_5) A_{5;1}(\bar{q}_1, q_2, g_3, g_4, g_5)\right]. \quad (6.23)$$

6.5 One-loop corrections to $\bar{q}q\overline{Q}Q\gamma$ amplitudes

In processes involving four quarks, one also obtains new color factors at one loop. However, for the one case of interest in this paper, the four-quark one-gluon amplitudes, the only new color structure involves the trace of the lone gluon generator and therefore vanishes. The authors of ref. [18] do not provide the partial amplitude that would have accompanied it. However, Signer [16] has given the one-loop four-quark one-photon amplitudes explicitly.

The color decomposition of the four-quark, one-photon amplitudes at one-loop is the same as at tree level, eq. (2.14),

$$\begin{aligned}
&\mathcal{A}_5^{\text{1-loop}}(\bar{q}_1, q_2, \overline{Q}_3, Q_4, \gamma_5) \quad (6.24) \\
&= \sqrt{2} e g^4 \left[\delta_{i_4}^{\bar{\imath}_1}\ \delta_{i_2}^{\bar{\imath}_3}\ A_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) - \frac{1}{N_c} \delta_{i_2}^{\bar{\imath}_1}\ \delta_{i_4}^{\bar{\imath}_3}\ B_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \right],
\end{aligned}$$

with

$$\begin{aligned} A_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) &= Q_q u_2^{(1)}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) + Q_Q d_2^{(1)}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \\ B_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) &= Q_q u_1^{(1)}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) + Q_Q d_1^{(1)}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \,. \end{aligned} \tag{6.25}$$

The functions $u_i^{(1)}$

$$\begin{aligned} & u_1^{(1)}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \\ &= ic_\Gamma\, N_c \left[u_1^l(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) + \frac{1}{N_c^2}\, u_1^s(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) - \frac{n_f}{N_c} u^{n_f}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \right] \end{aligned} \tag{6.26}$$

$$\begin{aligned} & u_2^{(1)}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \\ &= ic_\Gamma\, N_c \left[u_2^l(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) + \frac{1}{N_c^2}\, u_2^s(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) + \frac{n_f}{N_c} u^{n_f}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \right] \end{aligned} \tag{6.27}$$

with functions $d_i^{(1)}$ in eq. (6.25) decomposed in the same way[5]. Our eq. (6.24) differs from its in equivalent in ref. [16] because we adopt the convention of refs. [15, 20] and define c_Γ to be

$$c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}} \frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \,. \tag{6.28}$$

This differs from the convention of ref. [16] by a factor of $1/(16\pi^2)$.

All of the functions on the right hand side of eqs. (6.26) - (6.27) and the corresponding functions $d_i^{(1)}$ are given explicitly in ref. [16]. The expressions given there, however, are written in terms of symbols which are only defined in ref. [18]. In general, these symbols represent recurring combinations of logarithms and dilogarithms of the kinematic invariants. It turns out that these symbols are in a one-to-one correspondence to a different set of symbols used by Bern, Dixon and Kosower in refs. [15, 20]. To facilitate the understanding of these expressions, we present a conversion table in Appendix C.

In evaluating the tree-level amplitude one finds that

$$B_5^{\text{tree}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) = A_5^{\text{tree}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5). \tag{6.29}$$

This degeneracy causes the $B_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5)$ term to drop out of the interference term, with the result that the one-loop correction to the cross section is

$$\sigma^{\text{1-loop}}_{\bar{q}q\overline{Q}Q\gamma} = 4e^2 g^6 (N_c^2-1) \sum_{\text{helicities}} \text{Re}\left[A_5^{\text{tree}*}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) A_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5) \right] . \tag{6.30}$$

Thus, $B_5^{\text{1-loop}}(\bar{q}_1, q_2; \overline{Q}_3, Q_4; \gamma_5)$ need not be computed for NLO calculations, but would be needed for inclusion in a NNLO calculation.

[5]Note that eq. (19) in ref. [16] (the equivalent of our eq. (6.26)) is missing a factor of N_c [19].

7 Conclusions

In this paper, we have computed compact expressions for the six-parton tree-level amplitudes involving up to three photons. In addition, we have presented detailed expansions for the corresponding five-parton one-loop amplitudes, in terms of known results. These are all of the amplitudes needed to compute at NLO two-parton to three-parton scattering where up to three partons are photons, and two-parton to two-parton inclusive scattering where three partons are photons. They also contribute at NNLO to two-parton to two-parton scattering where one or two partons are photons.

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A Two-quark, three-gluon, one-photon amplitudes

The two-quark, three-gluon, one-photon amplitudes of eqs. (3.1) - (3.3) can be computed from the two-quark, four-gluon amplitudes by summing over all the possible positions of a gluon in the color ordering. A more efficient procedure is to sum over the positions of a gluon of the corresponding two-gluino, four-gluon amplitude. The ensuing sum can then be simplified by using the dual Ward identities [14], and can be reformulated in terms of a sum of purely gluonic amplitudes and two-quark, four-gluon amplitudes by using supersymmetric Ward identities [14, 21]. For the configurations of eqs. (3.1) - (3.3), we obtain

$$
\begin{aligned}
A_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^-, g_3^+, \gamma_4^+) &= -\left(\frac{\langle 5\,1\rangle}{\langle 6\,1\rangle} + \frac{\langle 2\,1\rangle\,[4\,2]}{\langle 6\,1\rangle\,[4\,5]}\right) A_6^{\rm tree}(g_1^-, g_2^-, g_3^+, g_5^+, g_4^+, g_6^-) \\
&\quad -\frac{\langle 2\,1\rangle\,[4\,3]}{\langle 6\,1\rangle\,[4\,5]}\, A_6^{\rm tree}(\bar{q}_3^+, q_2^-, g_1^-, g_6^-, g_4^+, g_5^+) \\
A_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^+, g_2^-, g_3^-, \gamma_4^+) &= \\
-\frac{\langle 5\,2\rangle}{\langle 6\,2\rangle}\, A_6^{\rm tree}(g_1^+, g_2^-, g_3^-, g_5^+, g_4^+, g_6^-) &+ \frac{\langle 3\,2\rangle}{\langle 6\,2\rangle}\, A_6^{\rm tree}(\bar{q}_5^+, q_3^-, g_2^-, g_1^+, g_6^-, g_4^+) \\
A_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^+, g_3^-, \gamma_4^+) &= \\
-\frac{\langle 5\,1\rangle}{\langle 6\,1\rangle}\, A_6^{\rm tree}(g_1^-, g_2^+, g_3^-, g_5^+, g_4^+, g_6^-) &+ \frac{\langle 3\,1\rangle}{\langle 6\,1\rangle}\, A_6^{\rm tree}(\bar{q}_5^+, q_3^-, g_2^+, g_1^-, g_6^-, g_4^+)\,.
\end{aligned}
\tag{A.1}
$$

All the other configurations can be obtained from the ones above by parity inversion and/or charge conjugation.

B Two-quark, two-gluon, two-photon amplitudes

In converting two-quark, four-gluon amplitudes into two-quark, two-gluon, two-photon amplitudes, we follow the same procedure as in Appendix A. For the configurations of table 3, we have obtained

$$
\begin{aligned}
&A_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^-, \gamma_3^+, \gamma_4^+) = \\
&\quad \sum_{\sigma\in Z_2} \left(\frac{\langle 5\,1\rangle}{\langle 6\,1\rangle} A_6^{\rm tree}(g_1^-, g_2^-, g_5^+, g_{\sigma_3}^+, g_{\sigma_4}^+, g_6^-) - \frac{\langle 2\,1\rangle}{\langle 6\,1\rangle} A_6^{\rm tree}(\bar{q}_5^+, q_2^-, g_1^-, g_6^-, g_{\sigma_4}^+, g_{\sigma_3}^+) \right) \\
&A_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^+, g_2^-, \gamma_3^-, \gamma_4^+) = \\
&\quad \frac{\langle 5\,3\rangle}{\langle 6\,3\rangle} A_6^{\rm tree}(g_1^+, g_2^-, g_5^+, g_4^+, g_3^-, g_6^-) + \frac{\langle 5\,2\rangle}{\langle 6\,2\rangle} A_6^{\rm tree}(g_1^+, g_2^-, g_5^+, g_3^-, g_4^+, g_6^-) \\
&\quad - \frac{\langle 2\,3\rangle}{\langle 6\,3\rangle} A_6^{\rm tree}(\bar{q}_5^+, q_2^-, g_1^+, g_6^-, g_3^-, g_4^+) - \frac{\langle 3\,2\rangle}{\langle 6\,2\rangle} A_6^{\rm tree}(\bar{q}_5^+, q_3^-, g_4^+, g_6^-, g_1^+, g_2^-) \\
&A_6^{\rm tree}(\bar{q}_5^+, q_6^-, g_1^-, g_2^+, \gamma_3^-, \gamma_4^+) = \\
&\quad \left(\frac{\langle 5\,1\rangle}{\langle 6\,1\rangle} + \frac{\langle 3\,1\rangle\,[2\,3]}{\langle 6\,1\rangle\,[2\,5]} \right) A_6^{\rm tree}(g_1^-, g_2^+, g_5^+, g_4^+, g_3^-, g_6^-) + \frac{\langle 5\,1\rangle}{\langle 6\,1\rangle} A_6^{\rm tree}(g_1^-, g_2^+, g_5^+, g_3^-, g_4^+, g_6^-) \\
&\quad + \frac{\langle 3\,1\rangle\,[2\,4]}{\langle 6\,1\rangle\,[2\,5]} A_6^{\rm tree}(\bar{q}_4^+, q_3^-, g_6^-, g_1^-, g_2^+, g_5^+) - \frac{\langle 3\,1\rangle}{\langle 6\,1\rangle} A_6^{\rm tree}(\bar{q}_5^+, q_3^-, g_4^+, g_6^-, g_1^-, g_2^+) ,
\end{aligned}
\tag{B.1}
$$

where the sum in the first equation is over permutations of 3 and 4. All the other configurations can be obtained from the ones above by photon relabeling, parity inversion and/or charge conjugation.

C Translating symbols used in NLO matrix elements

The matrix elements for the four-quark one-photon process at one-loop are given in ref. [16] in terms of symbols which are defined in ref. [18]. These symbols have a one-to-one correspondence with symbols defined in ref. [15] that are used in the two-quark three-gluon primitives. To facilitate the reading and use of ref. [16], we provide a translation table for those symbols.

Signer (ref. [16])	Bern,*et. al.* (ref. [15])
$\left\{{ij \atop kl}\right\}_0$	$\ln\left(\frac{-s_{ij}}{-s_{kl}}\right)$
$\left\{{ij \atop kl}\right\}_1$	$\frac{1}{s_{kl}}\mathrm{L}_0\left(\frac{-s_{ij}}{-s_{kl}}\right)$
$\left\{{ij \atop kl}\right\}_2$	$\frac{1}{s_{kl}^2}\mathrm{L}_1\left(\frac{-s_{ij}}{-s_{kl}}\right)$
$\left\{{ij \atop kl}\right\}_3$	$\frac{1}{s_{kl}^3}\mathrm{L}_2\left(\frac{-s_{ij}}{-s_{kl}}\right)$
$\mathcal{F}(i,j,k)$	$\mathrm{Ls}_{-1}\left(\frac{-s_{ij}}{-s_{mn}},\ \frac{-s_{jk}}{-s_{mn}}\right)\Big\vert_{\{m,n\}=\{1,2,3,4,5\}/\{i,j,k\}}$
$\mathcal{P}_{ij}$	$\left(\frac{\mu^2}{-s_{ij}}\right)^{\epsilon}$
$\langle ijkl \rangle$	$\frac{\langle i\,k\rangle\,\langle j\,l\rangle}{\langle i\,l\rangle\,\langle j\,k\rangle}$

Table 5: Conversion table relating the undefined symbols in ref. [16] to their equivalents defined in ref. [15]

References

[1] CMS Collaboration, Technical Proposal, CERN/LHCC 94-38 (1994);
ATLAS Collaboration, Technical Proposal, CERN/LHCC 94-43 (1994);
CMS Collaboration, Technical Design Report, CERN/LHCC 97-33 (1997);
ATLAS Collaboration, Detector and Physics Performance Technical Design Report, Vol. II, CERN/LHCC 99-15 (1999).

[2] D. Graudenz, M. Spira and P. Zerwas, *Phys. Rev. Lett.* **70** (1993) 1372;
A. Djouadi, D. Graudenz, M. Spira and P. Zerwas, *Nucl. Phys.* **B453** (1995) 17.

[3] B. Bailey, J.F. Owens and J. Ohnemus, *Phys. Rev. D* **46** (1992) 2018;
B. Bailey and J.F. Owens, *Phys. Rev. D* **47** (1993) 2735.

[4] E. Pilon, presentation at the LHC Workshop, Les Houches, June 1999.

[5] D. de Florian and Z. Kunszt, *Phys. Lett.* **B460** (1999) 184 [`hep-ph/9905283`].

[6] C. Balazs, P. Nadolsky, C. Schmidt and C.P. Yuan, preprint `hep-ph/9905551`.

[7] S.Abdullin, et al., *Phys. Lett.* **B431** (1998) 410 [`hep-ph/9805341`].

[8] R.K. Ellis, I. Hinchliffe, M. Soldate and J.J. van der Bij, *Nucl. Phys.* **B297** (1988) 221.

[9] D. de Florian, M. Grazzini and Z. Kunszt, *Phys. Rev. Lett.* **82**, 5209 (1999) [`hep-ph/9902483`].

[10] ZEUS Collaboration, J.Breitweg et al., *Phys. Lett.* **B443** (1998) 394 [hep-ex/9810046].

[11] J.E. Paton and H.-M. Chan, *Nucl. Phys.* **B10** (1969) 516;
P. Cvitanovic, P.G. Lauwers, and P.N. Scharbach, *Nucl. Phys.* **B186** (1981) 165;
F.A. Berends and W.T. Giele, *Nucl. Phys.* **B294** (1987) 700;
D.A. Kosower, B.-H. Lee and V.P. Nair, *Phys. Lett.* **B201** (1988) 85;
M. Mangano, S. Parke and Z. Xu, *Nucl. Phys.* **B298** (1988) 653;
M. Mangano, *Nucl. Phys.* **B309** (1988) 461;
D. Zeppenfeld, Int. J. *Mod. Phys.* **A3** (1988) 2175;
Z. Bern and D.A. Kosower, Nucl. Phys. **B362** (1991) 389.

[12] Z. Bern and G. Chalmers, *Nucl. Phys.* **B447**, 465 (1995) [hep-ph/9503236];
Z. Bern, V. Del Duca and C.R. Schmidt, *Phys. Lett.* **B445**, 168 (1998) [hep-ph/9810409];
Z. Bern, V. Del Duca, W.B. Kilgore and C.R. Schmidt, preprint [hep-ph/9903516];
D.A. Kosower and P. Uwer, preprint [hep-ph/9903515].

[13] R. Gastmans and T.T. Wu, *The ubiquitous photon: Helicity Method for QED and QCD*, Clarendon Press, 1990.

[14] M. Mangano and S.J. Parke, *Phys. Rep.* **200**, 301 (1991).

[15] Z. Bern, L. Dixon and D.A. Kosower, *Nucl. Phys.* **B437** (1995) 259 [hep-ph/9409393].

[16] A. Signer, *Phys. Lett.* **B357** (1995) 204 [hep-ph/9507442].

[17] CALKUL Collaboration, F.A. Berends et al., *Nucl. Phys.* **B239** (1984) 395.

[18] Z. Kunszt, A. Signer and Z. Trócsányi, *Phys. Lett.* **B336** (1994) 529 [hep-ph/9405386].

[19] A. Signer, private communication.

[20] Z. Bern, L. Dixon and D.A. Kosower, *Phys. Rev. Lett.* **70** (1993) 2677 [hep-ph/9302280].

[21] M.T. Grisaru, H.N. Pendleton and P. van Nieuwenhuizen, *Phys. Rev.* **D15** (1977) 996;
M.T. Grisaru and H.N. Pendleton, *Nucl. Phys.* **B124** (1977) 81;
S.J. Parke and T. Taylor, *Phys. Lett.* **B157** (1985) 81.